Exercises for lecture 10

Consider the following density function.

$$p(x) \propto \tilde{p}(x) = \exp(-0.5x^2)\left(4\sin^2(6x) + 3\cos^2(x)\sin^2(4x) + 1\right) \tag{1}$$

Assuming that we do not know this analytical form of the density but we can evaluate it at any point we like, we would like to sample from $p(x)$ using different methods.

1. Use rejection sampling to obtain 1000 samples from $p(x)$. Consider two different proposals:

   (i) Uniform distribution $q(x) = \mathcal{U}(x; -4, 4)$.

   (ii) Gaussian distribution $q(x) = \mathcal{N}(x; 0, 1)$.

   Note that for both cases, you should find the constants K that satisfy $\tilde{p}(x) \leq Kq(x)$ for all x. Assume that $p(x) = \tilde{p}(x) = 0$ outside the interval $[-4, 4]$ for the uniform case.

2. Use the function `KernelDensityEstimate.m` supplied with the exercise on the course web-page to construct a density estimate for $p(\cdot)$ for both cases from the samples. See the help of the function. You can choose the points to evaluate the density as $x = -4 : 0.01 : 4$ and the kernel standard deviation as 0.1.

3. Use the Metropolis–Hastings algorithm to sample from $p(x)$. You can use the proposal density $q(x|z) = \mathcal{N}(x; z, 0.01^2)$. Note that sampling from this density can be done by adding the variable z (previous sample in M-H algorithm) and a random number $w_k \sim \mathcal{N}(w_k; 0, 0.01^2)$. Start your algorithm from the sample $x^{(0)} = -2$ and observe how the algorithm behaves. Try increasing and decreasing the standard deviation of the proposal and see how the behavior changes. When you think that enough samples are obtained, construct the kernel based density estimate from the samples as you did in Exercise-2.

4. Obtain (one of) the globally maximizing point(s) (or a close approximation) for $p(\cdot)$ using the simulated annealing algorithm. For this purpose, you can adapt the M-H algorithm you designed in the previous exercise. At each step keep the best point (with the highest value of $\tilde{p}$) achieved in memory. Plot the best point achieved and maximum function value achieved w.r.t. the iteration number as was done in the lecture notes.

References

[1] Robert C. P.; Casella G., Monte Carlo Statistical Methods, Springer, 2004.