Supervisory Control of Hybrid Systems

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Notes

- Try to explain the concept of supervisory control
- Examples: thermostat
- A continous system is controlled with a discrete process, e.g. digital control

The way to go...

- Hybrid modelling
- Discrete Event System model
  - model
  - properties
- Interface
- Supervisory Control Design
  - Hybrid systems
  - Controllability

Notes

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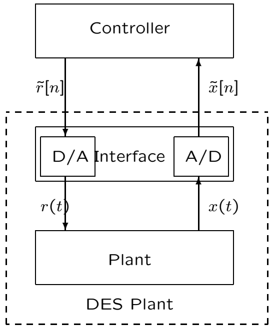
Hybrid modelling

- **Continuous plant: $\dot{x}(t) = f(x(t), r(t))$**
- **Controller: $S = (\tilde{S}, \tilde{X}, \tilde{R}, \delta, \phi)$**
- **Interface:**
  - **generator: A/D or state to symbol**
  - **actuator: D/A or control symbol to input signal**

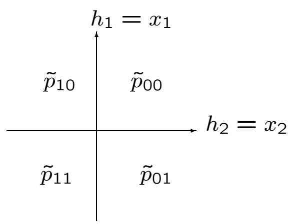


Notes

- **nonlinear plant**
- **DFA models controller**
- **actuator makes the control input piecewise constant**
- **generator consists of triggering mechanism based on *plant events*. In hybrid control the plant event occurs when crossing a hypersurface.**
- **Plant event is considered to be a realisation of a specified condition. (An area in the state space)**

DES model I

- **plant + actuator + generator ≈ DES plant model**
- **States in DES plant model is based upon hypersurfaces in generator.**
- $\tilde{p}_b = \{\xi \in R^n : b_i = 0 \Rightarrow h_i(\xi) > 0$ and $b_i = 1 \Rightarrow h_i(\xi) < 0\}$



Notes

- **Discrete event system**
- **Approximate the continous plant with a DES plant model,**
- **e.g. thermostat: 3 states. $(T < 20)$, $(20 \leq T \leq 25)$, $(T > 25)$. 2 plant events: $(x_2 \to x_1)$, $(x_2 \to x_3)$. Control: heater on when temp decreases below 20 (event 1), heater off when temp exceeds 25 (event 2).**
- **Hypersurfaces divides the continuous state space. The DES states are the equivalence classes defined by the hypersurfaces.**

DES model II

- Define properties for the model
  - Adjacent states at $(i \in I, \xi \in N(h_i))$
  - $\tilde{p}_c \in \varphi(\tilde{p}_b, \tilde{r}_k)$ iff $\exists$ a point where $\tilde{p}_c$ and $\tilde{p}_b$ are adjacent and where $\dot{x} = f(x, \gamma(\tilde{r}_k))$ is leaving $\tilde{p}_b$.
- DES plant model is a conservative approximation
- Hard to make deterministic

Notes

- possible to state mathematically
- $r(t) = \gamma(\tilde{r}[k])$, $k < t < k+1$
- notice the nondeterminism: $\tilde{p}_c \in \varphi(\tilde{p}_b, \tilde{r}_k)$
- actual behavior is included in the approximated behavior $\Rightarrow$ we can eliminate undesirable behavior but not permit desirable behavior in the actual system.

Interface

- Generator design
  - Assume state partition fixed
  - Identify target region, T, and starting region, S, for a given control goal.
  - Find all the states that can be driven to T using one control policy. Until S is contained in this set of states, F, let T′=T∪F and repeat for the target region T′.
- Difficult to find the set of states, adressed by finding subsets called “common flow regions”.

Notes

- Interface is important when determining the dynamics of the hybrid control system.
- Common flow regions are bounded by invariant manifolds and an exit boundary. The state trajectory can only leave/enter the region through the exit boundary.
- Define transition-stability. If we start a little outside the starting region we will still get to the same state and receive the same output symbol if we use a specific control policy.

Supervisory control design

- Generated language is used to describe performance specifications. The language consists of strings of plant event symbols.
- Objective is to build a supervisor/controller s.t. the closed-loop language is contained in a desired language K. K describes which states that are allowed and which ar not.
- A controllable language means that if the current output string is in the language we can find a control symbol that will keep us in hte language, i.e. undesirable states/strings can be avoided.

Notes

- The language considered is the one produced by the DES plant model.
- If the desired language is not controllable find the maximal sublanguage that is, $K^{\uparrow}$.
- This only excludes states, since prefix-closure $\Leftrightarrow$ no final/accepting states. We cannot promise that we reach a specific state.