Target Tracking
Le 4: Multi Target Tracking: single hypothesis tracking

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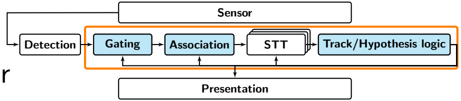
Summary: lecture 3

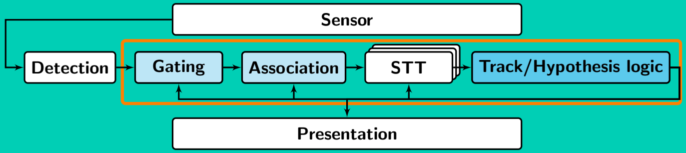


- Gate to improve complexity in presence of clutter
  - Rectangular: cheap but crude
  - Ellipsoidal: more correct
- Track logic determines if there is an object present of not
  - State-machine for confirming target, based on gated measurements
  - Score based logic, based on a hypothesis test
- Different association strategies exist (so far for STT)
  - Nearest neighbor (NN) association
    A hard decision to use the "closest" measurement.
  - Probabilistic data association (PDA)
    A soft decision where all measurements in the gate are combined.

References on Multiple Target Tracking Topics

- D. Bertsekas. Auction algorithms.
  URL http://www.mit.edu/~dimitrib/Auction_Encycl.pdf (Auction algorithm)
- B.-N. Vo, M. Mallick, Y. Bar-Shalom, S. Coraluppi, R. Osborne, III, R. Mahler, and B.-T. Vo.
  Multitarget Tracking.
  Wiley Encyclopedia of Electrical and Electronics Engineering, 2015.
  URL https://www.researchgate.net/publication/283623828_Multitarget_Tracking
  (MTT, GNN)
- Y. Bar-Shalom, F. Daum, and J. Huang. The probabilistic data association filter.
  IEEE Control Systems Magazine, 29(6):82–100, Nov. 2009. (PDA/JPDA)

Multi-Target Tracking



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Association: a multi target tracking perspective

Definition: association

Association is the process of assigning measurements to existing tracks or existing tracks to measurements (measurement-to-track association *vs.* track-to-measurement association).

- In the classical air traffic control (ATC) application, there are hundreds of targets and measurements.
- The number of possible combinations of measurements and targets grows combinatorally.
- Not all associations are likely or even feasible.
- Very unlikely combinations should be removed as possible!

Association Hypothesis

Definition: association hypothesis

An (association) **hypothesis** is a partitioning of a set of measurements according to the their origin; individual existing targets, clutter/false detections, and new targets.

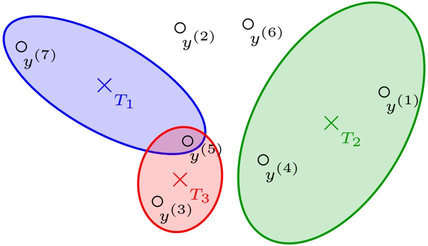
- A **single hypothesis tracker** (SHT) maintains a *single* hypothesis about all of the measurements received over time.
  - The global nearest neighbor (GNN) algorithm does this by selecting the best hypothesis according to a criterion.
  - The joint probabilistic data association (JPDA) filter combines all possible current hypotheses into a single hypotesis.
- A **multiple hypothesis** tracker (MHT), maintains *multiple* hypotheses about the origin of the received measurements.

Multi Target Associaion: example

- Using STT for each target, results in locally optimal solutions, which might be infeasible.

  Consider the associations: $T_1 \leftrightarrow y^{(5)}$, $T_2 \leftrightarrow y^{(1)}$, $T_3 \leftrightarrow y^{(5)}$ *which picks the best measurement for each target, but violates the assumption that a measurement originates from a single target.*

- In MTT the complete association hypothesis is considered, to only obtain a global optimum and avoid infeasible solutions.



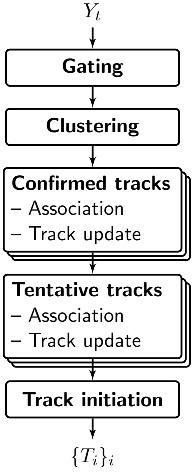
Example with three targets, $T_1, \ldots, T_3$*, and seven measurements* $y^{(1)}, \ldots, y^{(7)}$*.*

Track logic and gating will be utilized to simplify the MTT process.

Single Hypothesis Tracking

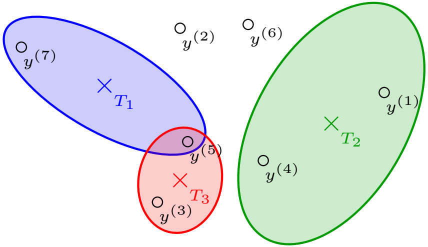
Principal steps:

1. Gating
   Gating is performed, yielding a validation matrix $\mathcal{V}$ indicating with measurements should be considered for each track.
2. Clustering
   Tracks that do not share potential measurements are separated, yielding many smaller problems.
3. Association and updating of confirmed tracks
   Associate measurements to confirmed tracks and update the tracks. From now on, do not consider any measurements that has been gated with a confirmed track.
4. Association and updating of tentative tracks
   Update the procedure with the remaining measurements and the tentative tracks.
5. Initiate new tentative tracks
   Use remaining measurements to start tentative tracks.



Gating and Validation Matrix

- Perform gating between all measurements and targets (using suitable gating strategy)
- Create the validation matrix $\mathcal{V}$, where each element indicate if the measurement and track are compatible or not.
- The validation matrix is used to create the assignment hypothesis.



	T_1	T_2	T_3
$y^{(1)}$	0	1	0
$y^{(2)}$	0	0	0
$y^{(3)}$	0	0	1
$y^{(4)}$	0	1	0
$y^{(5)}$	1	0	1
$y^{(6)}$	0	0	0
$y^{(7)}$	1	0	0

Validation matrix, $\mathcal{V}$

Example of gating and resulting validation matrix

Assignment: notation

Measurement origins

If we consider measurements in a scan and existing tracks:

TC Track Continuation: a measurement will update a track

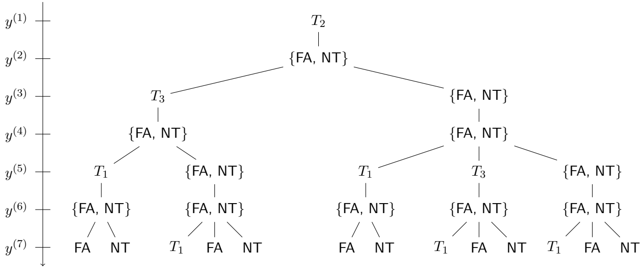
FA False Alarm: a measurement is considered as nuisance

NT New Track: a measurement can start a new track

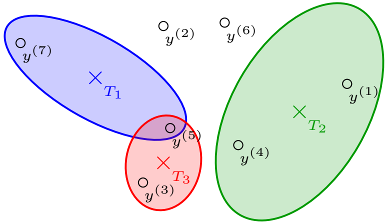
It is reasonable to assume that a measurement can only be used for one of the above.

Possible Association Hypotheses

Ex: consider the case where $y_t^{(1)}$ is associated to T_2.



To complete: repeat for $y_t^{(1)} = \text{FA}$ and $y_t^{(1)} = \text{NT}$, and {FA, NT} indicates that FA and NT yields identical subtrees.



	T_1	T_2	T_3
$y^{(1)}$	0	1	0
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$y^{(3)}$	0	0	1
$y^{(4)}$	0	1	0
$y^{(5)}$	1	0	1
$y^{(6)}$	0	0	0
$y^{(7)}$	1	0	0

Validation matrix, $\mathcal{V}$

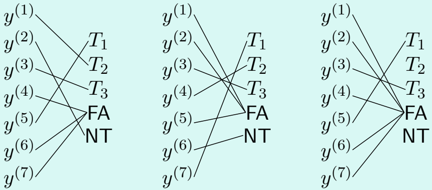
Association Hypothesis: example

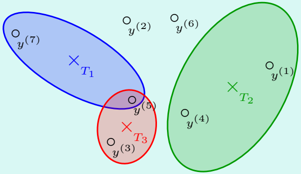
Define the **association hypothesis** θ_t as a mapping

$$\theta_t(\cdot) : \{1, 2, \ldots, m_t\} \rightarrow \{\mathsf{FA}, 1, 2, \ldots, n_t, \mathsf{NT}\}$$

- m_t is the number of measurements in (scan) Y_t, *i.e.*, $Y_t = \{y_t^{(1)}, \ldots, y_t^{(m_t)}\}$
- n_t is the number of tracks when entering the frame.

Example: hypotheses when $m_t = 7$, $n_t = 3$





	T_1	T_2	T_3
$y^{(1)}$	0	1	0
$y^{(2)}$	0	0	0
$y^{(3)}$	0	0	1
$y^{(4)}$	0	1	0
$y^{(5)}$	1	0	1
$y^{(6)}$	0	0	0
$y^{(7)}$	1	0	0

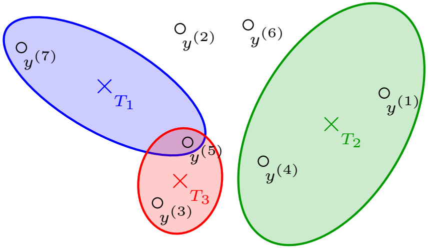
Validation matrix, $\mathcal{V}$

Clustering

- Computational complexity scales exponentially the with number of measurements and targets.
- Tracks that do not share any measurements can be treated separately, to reduce the complexity.
- Clusters in the example: $\mathcal{C}^{(1)} = \{T_1, T_3\}$, $\mathcal{C}^{(2)} = \{T_2\}$.

	T_1	T_2	T_3
$y^{(1)}$	0	1	0
$y^{(2)}$	0	0	0
$y^{(3)}$	0	0	1
$y^{(4)}$	0	1	0
$y^{(5)}$	1	0	1
$y^{(6)}$	0	0	0
$y^{(7)}$	1	0	0

Validation matrix, $\mathcal{V}$



Clustering

- Computational complexity scales exponentially the with number of measurements and targets.
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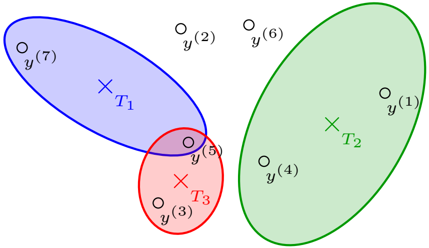
- Clusters in the example: $\mathcal{C}^{(1)} = \{T_1, T_3\}$, $\mathcal{C}^{(2)} = \{T_2\}$.

	T_1	T_3
$y^{(3)}$	0	1
$y^{(5)}$	1	1
$y^{(7)}$	1	0

Validation matrix, $\mathcal{V}^{(1)}$

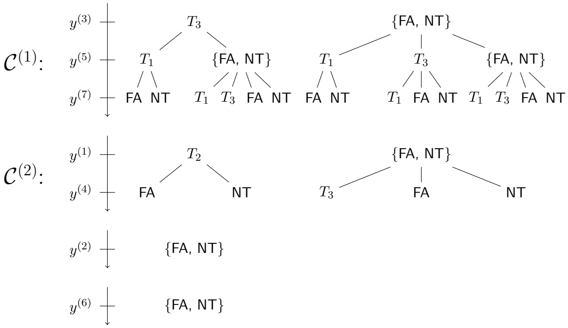
	T_2
$y^{(1)}$	1
$y^{(4)}$	1

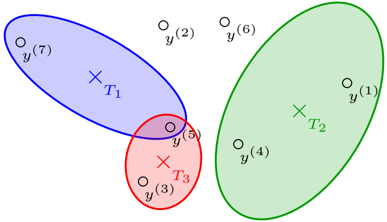
Validation matrix, $\mathcal{V}^{(2)}$



$y^{(2)}$ and $y^{(6)}$ do not fit any gate and can only be FA or NT.

Association Hypotheses: revisited using clustering





	T_1	T_3
$y^{(3)}$	0	1
$y^{(5)}$	1	1
$y^{(7)}$	1	0

Validation matrix, $\mathcal{V}^{(1)}$

	T_2
$y^{(1)}$	1
$y^{(4)}$	1

Validation matrix, $\mathcal{V}^{(2)}$

The selections in the respective clusters can be made independently!

Hypothesis Probabilities: track continuation

Track Continuation (TC)

- Detection probability: P_{D}
- Gate probability: P_{G}
- Predicted measurement density of jth target: $p^{(j)}_{t|t-1}(y)$.

In the KF case:

$$p^{(j)}_{t|t-1}(y) = \mathcal{N}(y; \hat{y}^{(j)}_{t|t-1}, S^{(j)}_{t|t-1})$$

Hypothesis Probabilities: false alarm

False alarm (FA)

- Number of false alarms, m_t^{FA}, in V is distributed as:

$$P_{\text{FA}}(m_t^{\text{FA}}) = \frac{(\beta_{\text{FA}} V)^{m_t^{\text{FA}}} e^{-\beta_{\text{FA}} V}}{m_t^{\text{FA}}!}$$

- False alarm spatial density is $p_{\text{FA}}(y) = 1/V$

Hypothesis Probabilities: new track

New Target (NT)

- Number of new targets, m_t^{NT} is distributed as

$$P_{\mathrm{NT}}(m_t^{\mathrm{NT}}) = \frac{(\beta_{\mathrm{NT}} V)^{m_t^{\mathrm{NT}}} e^{-\beta_{\mathrm{NT}} V}}{m_t^{\mathrm{NT}}!}$$

- New target spatial density is $p_{\mathrm{NT}}(y) = 1/V$

Hypothesis Probabilities: FA and NT

Let $\mathcal{J}^{\text{FA}}$ be the set of false alarms (with m_t^{FA} elements), then

$$\Pr(\mathcal{J}^{\text{FA}} \text{are the FA}) = m_t^{\text{FA}}! P_{\text{FA}}(m_t^{\text{FA}}) \prod_{i \in \mathcal{J}^{\text{FA}}} p_{\text{FA}}(y_t^{(i)}).$$

The FA are unordered, hence $m_t^{\text{FA}}!$ compensates for all the FA association possibilities. Insert Poisson distributed clutter uniformly in the tracking volume:

$$\Pr(\mathcal{J}^{\text{FA}} \text{are the FA}) = m_t^{\text{FA}}! \frac{(\beta_{\text{FA}} V_t)^{m_t^{\text{FA}}} e^{-\beta_{\text{FA}} V_t}}{m_t^{\text{FA}}!} \frac{1}{V_t^{m_t^{\text{FA}}}} \propto (\beta_{\text{FA}})^{m_t^{\text{FA}}}$$

Hypothesis Probabilities: FA and NT

Let $\mathcal{J}^{\text{FA}}$ be the set of false alarms (with m_t^{FA} elements), then

$$\Pr(\mathcal{J}^{\text{FA}}\text{are the FA}) = m_t^{\text{FA}}! P_{\text{FA}}(m_t^{\text{FA}}) \prod_{i \in \mathcal{J}^{\text{FA}}} p_{\text{FA}}(y_t^{(i)}).$$

The FA are unordered, hence $m_t^{\text{FA}}!$ compensates for all the FA association possibilities. Insert Poisson distributed clutter uniformly in the tracking volume:

$$\Pr(\mathcal{J}^{\text{FA}}\text{are the FA}) = m_t^{\text{FA}}! \frac{(\beta_{\text{FA}} V_t)^{m_t^{\text{FA}}} e^{-\beta_{\text{FA}} V_t}}{m_t^{\text{FA}}!} \frac{1}{V_t^{m_t^{\text{FA}}}} \propto (\beta_{\text{FA}})^{m_t^{\text{FA}}}$$

The NT case follows analogously.

Hypothesis Probabilities: putting it all together (1/2)

Consider association hypotesis θ_t in measurement scan Y_t.

$$P(\theta_t|Y_t) \propto (\beta_{\text{FA}})^{m_t^{\text{FA}}} (\beta_{\text{NT}})^{m_t^{\text{NT}}} \Big[\prod_{j\in\mathcal{J}} P_{\text{D}} p_{t|t-1}^{(j)}\big(y_t^{(\theta_t^{-1}(j))}\big)\Big] \Big[\prod_{j\in\bar{\mathcal{J}}} (1 - P_{\text{D}} P_{\text{G}})\Big],$$

where

- $\mathcal{J}$ is the set of indices of detected tracks (assigned).
- $\bar{\mathcal{J}}$ is the set of indices of non-detected tracks (not assigned).
- $\theta_t^{-1}(j)$ is the index of the measurement that is assigned to track $j \in \mathcal{J}$.
  ($\theta_t^{-1}(j) = \emptyset$ is shorthand for no measurement associated with track j.)
- All but the last factors are associated with a measurement.

Hypothesis Probabilities: putting it all together (2/2)

The association is simplified total probability can is a combination of measurement contributions, hence

$$
\begin{aligned}
P(\theta_t|Y_t) &\propto (\beta_{\text{FA}})^{m_t^{\text{FA}}}(\beta_{\text{NT}})^{m_t^{\text{NT}}}\Big[\prod_{j\in\mathcal{J}} P_{\text{D}} p^{(j)}_{t|t-1}\big(y_t^{(\theta_t^{-1}(j))}\big)\Big]\Big[\prod_{j\in\bar{\mathcal{J}}}(1-P_{\text{D}}P_{\text{G}})\Big] \\
&= \beta_{\text{FA}}^{m_t^{\text{FA}}}\beta_{\text{NT}}^{m_t^{\text{NT}}}\Big[\prod_{j\in\mathcal{J}}\frac{P_{\text{D}} p^{(j)}_{t|t-1}\big(y_t^{\theta_t^{-1}(j)}\big)}{(1-P_{\text{D}}P_{\text{G}})}\Big]\Big[\prod_{j\in\bar{\mathcal{J}}\cup\mathcal{J}}(1-P_{\text{D}}P_{\text{G}})\Big] \\
&= \beta_{\text{FA}}^{m_t^{\text{FA}}}\beta_{\text{NT}}^{m_t^{\text{NT}}}\Big[\prod_{j\in\mathcal{J}}\frac{P_{\text{D}} p^{(j)}_{t|t-1}\big(y_t^{\theta_t^{-1}(j)}\big)}{(1-P_{\text{D}}P_{\text{G}})}\Big]\Big(1-P_{\text{D}}P_{\text{G}}\Big)^{m_t} \\
&\propto \beta_{\text{FA}}^{m_t^{\text{FA}}}\beta_{\text{NT}}^{m_t^{\text{NT}}}\Big[\prod_{j\in\mathcal{J}}\frac{P_{\text{D}} p^{(j)}_{t|t-1}\big(y_t^{\theta_t^{-1}(j)}\big)}{(1-P_{\text{D}}P_{\text{G}})}\Big]
\end{aligned}
$$

Hypothesis Probabilities: final logarithmic expression

Global logarithmic association proabiblity

$$\log P(\theta_t|Y_t) = m_t^{\mathrm{FA}} \log \beta_{\mathrm{FA}} + m_t^{\mathrm{NT}} \log \beta_{\mathrm{NT}} + \sum_{j\in\mathcal{J}} \log \frac{P_{\mathrm{D}} p_{t|t-1}^{(j)}\big(y_t^{(\theta_t^{-1}(j))}\big)}{(1 - P_{\mathrm{D}} P_{\mathrm{G}})}$$

Properties:

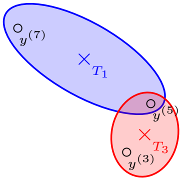
- One term per measurement
- The best association hence boils down to picking the right contribution from each measurement, in a consistent way

Assignment Matrix

The assignment matrix organizes the possible measurement contributions to $\log p(\theta_t|Y_t)$ in an efficient way.

	T_1	T_3	FA$_3$	FA$_5$	FA$_7$	NT$_3$	NT$_5$	NT$_7$
$y_t^{(3)}$	$-\infty$	ℓ_{33}	$\log \beta_{\text{FA}}$	$-\infty$	$-\infty$	$\log \beta_{\text{NT}}$	$-\infty$	$-\infty$
$y_t^{(5)}$	ℓ_{51}	ℓ_{53}	$-\infty$	$\log \beta_{\text{FA}}$	$-\infty$	$-\infty$	$\log \beta_{\text{NT}}$	$-\infty$
$y_t^{(7)}$	ℓ_{71}	$-\infty$	$-\infty$	$-\infty$	$\log \beta_{\text{FA}}$	$-\infty$	$-\infty$	$\log \beta_{\text{NT}}$

Association matrix, $\mathcal{A}^{(1)}$



- The gain from assigning measurement $y^{(i)}$ to track T_j is

$$\ell_{ij} = \log \frac{P_{\text{D}} p_{t|t-1}^{(j)}(y_t^{(i)})}{(1 - P_{\text{D}} P_{\text{G}})}.$$

	T_1	T_3
$y^{(3)}$	0	1
$y^{(5)}$	1	1
$y^{(7)}$	1	0

Validation matrix, $\mathcal{V}^{(1)}$

Assignment Problem

Assume a scan with m measurements and n "track hypothesis" (TC, FA, NT).

- Given the matrix $\mathcal{A} \in \mathbb{R}^{m \times n}$ with $n \geq m$.
- Define the binary matrix $Z = [z_{ij}]$, with $z_{ij} \in \{0, 1\}$.

Problem definition

$$\begin{aligned} \underset{Z}{\text{maximize:}} \quad & \sum\nolimits_{i,j} z_{ij} \mathcal{A}_{ij} \\ \text{subject to:} \quad & \sum\nolimits_{j} z_{ij} = 1 \quad \forall i && (\dagger) \\ & \sum\nolimits_{i} z_{ij} \leq 1 \quad \forall j && (\ddagger) \end{aligned}$$

† Each measurement is associated to exactly one track/FA/NT.

‡ Each track/FA/NT is associated to at most one measurement.

This problem is called as **assignment problem** in optimization literature.

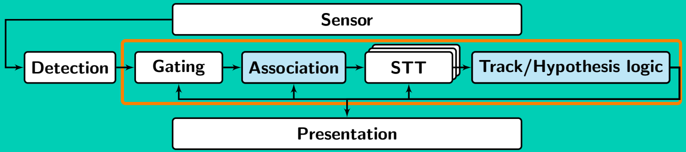
Assignment Problem: algorithms

- First considered in economics.
- For smaller problems an exhaustive search is possible, but this is inefficient.
- Earlier methods used linear programming techniques, like the Hungarian method which is computationally costly.

Assignment Problem: famous solutions

- Munkres algorithm obtains an optimal solution to the GNN assignment problem. An optimal solution minimizes the total cost of the assignments.
- Auction algorithm (by Bertsekas) finds a suboptimal solution to the GNN assignment problem by minimizing the total cost of assignment. While suboptimal, the auction algorithm is faster than the Munkres algorithm for large GNN assignment problems, for example, when there are more than 50 rows and columns in the cost matrix.
- JVC algorithm (by Jonker and Volgenant) solves the GNN assignment in two phases: begin with the auction algorithm and end with the Dijkstra shortest path algorithm.

Global Nearest Neighbor Tracker



Global Nearest Neighbor (GNN)

In each scan:

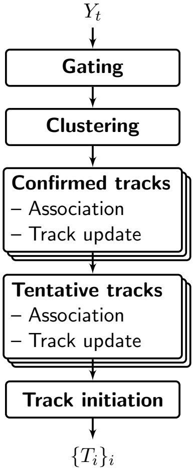
- Select the best association hypothesis, θ_t.
- Given θ_t:
  - Update all tracks with the associated measurement (usually using an EKF).
  - Update the track logic.
- Initiate new tracks from NT measurements.

Note on NT and FA handling

In the above steps, NT or FA does not matter, until the last step where anyhow all unassociated measurements should be given the chance to start up a new track.

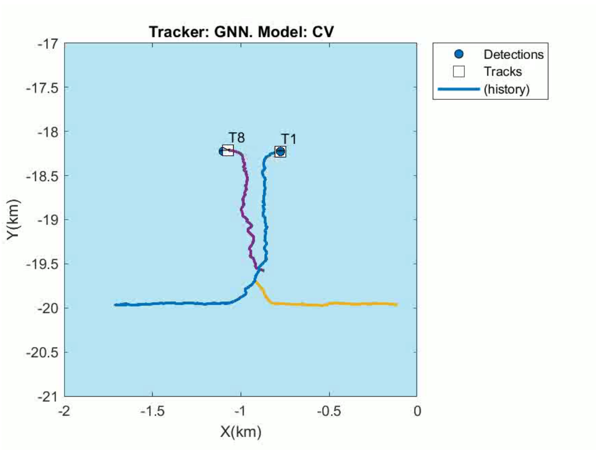
Introduce **external sources** (EX) combining FA and NT. EX becomes Poisson distributed with $\beta_{\text{EX}} = \beta_{\text{FA}} + \beta_{\text{NT}}$.

Global Nearest Neighbor (GNN): implementation details



- Apply gating and clustering to minimize the computational complexity.
- Use the EX trick to simplify the assignment problem further.
- Combine the tracking filter and target logic in one structure.
- Have separate containers for confirmed and tentative tracks.

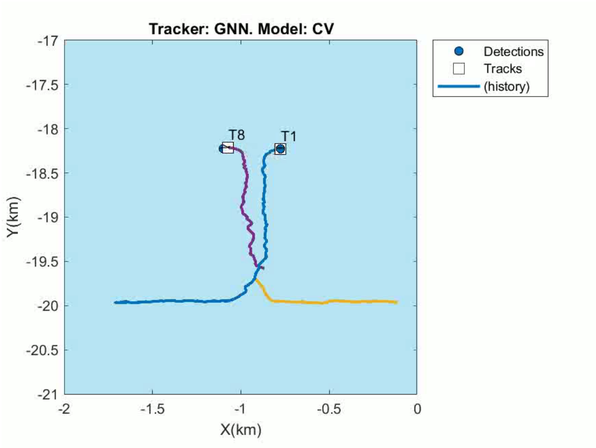
MTT: GNN CV-model



https://youtu.be/WPA2z-kw1wg

- *Global nearest neighbor* (GNN) tracker
- Simple *constant velocity* (CV) model
- Note the label switch and that one of the tracks is lost half way, and restarted as a new one.

MTT: GNN CV-model



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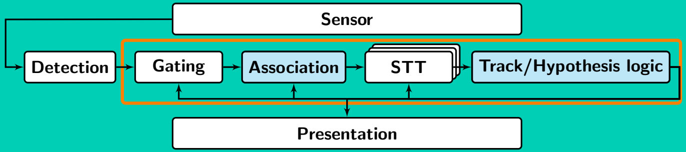
- *Global nearest neighbor* (GNN) tracker
- Simple *constant velocity* (CV) model
- Note the label switch and that one of the tracks is lost half way, and restarted as a new one.

Global Nearest Neighbor: properties

- Makes a hard association decision:
  - + Optimal when the correct association is made.
  - − Could break down completely with the wrong association.
- Works well when targets are well separated!
- Should not be used with poorly separated targets.
- Heavy clutter and low P_{D} could cause problems.
- Relatively fast and easy to implement.
- Works directly with the track logic discussed earlier.

JPDA



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Joint Probabilistic Data Association (JPDA) Filter

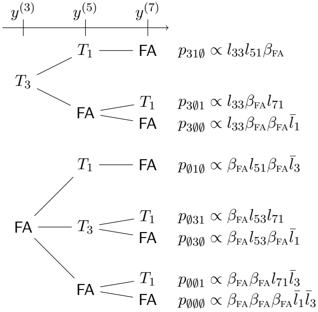
- JPDA is the soft decision equivalent of GNN in the way that PDA is a soft version of NN.
- All past is summarized with a single merged hypothesis.
- The number of targets is assumed fixed in the association, hence no NT associations in the possible hypotheses.
- A separate track initiation logic must run along with JPDAF to detect and initiate new tracks.

Joint Probabilistic Data Association (JPDA) Filter: details

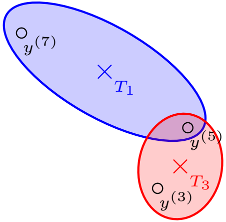
- All measurement associations are combined weighted with their likelihood of being true.
- For each previously established target, we need to calculate:
  - $P(\theta^{-1}(j) = i)$: Track T_j is associate measurement $y^{(i)}$.
  - $P(\theta^{-1}(j) = \emptyset)$: shorthand for no measurement is associated to T_j.
- For measurement $y_t^{(i)}$ in the gate, the update is then made using the PDA update formulas with slightly modified probabilities to account for global association consistency.

Joint Probabilistic Data Association: probabilities (1/2)

Enumerate all possible measurement hypotheses and compute their respective likelihood. This can be done for each cluster independently.



- $l_{ij} = P_{\text{D}} p^{(j)}_{t|t-1}(y^{(i)}_t)$
- $\bar{l}_j = 1 - P_{\text{G}} P_{\text{D}}$



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$y_t^{(3)}$	$-\infty$	ℓ_{33}	$\log \beta_{\text{FA}}$	$-\infty$	$-\infty$
$y_t^{(5)}$	ℓ_{51}	ℓ_{53}	$-\infty$	$\log \beta_{\text{FA}}$	$-\infty$
$y_t^{(7)}$	ℓ_{71}	$-\infty$	$-\infty$	$-\infty$	$\log \beta_{\text{FA}}$

Association matrix, $\mathcal{A}^{(1)}$

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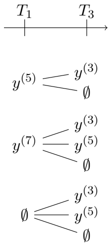
Joint Probabilistic Data Association: probabilities (2/2)

Rearrange the hypotheses to be able to compute the probability for each separate track.



$$
\begin{aligned}
p_{31\emptyset} &\propto \ell_{51}\ell_{33}\beta_{\text{FA}} \\
p_{\emptyset 1\emptyset} &\propto \ell_{51}\bar{\ell}_{3}\beta_{\text{FA}}^2 \\
p_{3\emptyset 1} &\propto \ell_{17}\ell_{33}\beta_{\text{FA}} \\
p_{\emptyset 31} &\propto \ell_{17}\ell_{53}\beta_{\text{FA}} \\
p_{\emptyset\emptyset 1} &\propto \ell_{17}\bar{\ell}_{3}\beta_{\text{FA}}^2 \\
p_{3\emptyset\emptyset} &\propto \bar{\ell}_{1}\ell_{33}\beta_{\text{FA}} \\
p_{\emptyset 3\emptyset} &\propto \bar{\ell}_{1}\ell_{53}\beta_{\text{FA}} \\
p_{\emptyset\emptyset\emptyset} &\propto \bar{\ell}_{1}\bar{\ell}_{3}\beta_{\text{FA}}^3
\end{aligned}
$$

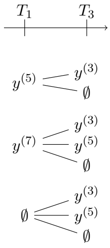
$$
\begin{aligned}
\Pr(\theta^{-1}(1) = 5) &= \tfrac{1}{C}(p_{31\emptyset} + p_{\emptyset 1\emptyset}) \\
\Pr(\theta^{-1}(1) = 7) &= \tfrac{1}{C}(p_{3\emptyset 1} + p_{\emptyset 31} + p_{\emptyset\emptyset 1}) \\
\Pr(\theta^{-1}(1) = \emptyset) &= \tfrac{1}{C}(p_{3\emptyset\emptyset} + p_{\emptyset 3\emptyset} + p_{\emptyset\emptyset\emptyset})
\end{aligned}
$$

- $l_{ij} = P_{\text{D}} p_{t|t-1}^{(j)}(y_t^{(i)})$
- $\bar{l}_j = 1 - P_{\text{G}} P_{\text{D}}$

$$C = \sum_i p_i$$

Joint Probabilistic Data Association: probabilities (2/2)

Rearrange the hypotheses to be able to compute the probability for each separate track.



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\begin{aligned}
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p_{\emptyset 3\emptyset} &\propto \bar{\ell}_1\ell_{53}\beta_{\text{FA}} \\
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$$

- $l_{ij} = P_{\text{D}} p_{t|t-1}^{(j)}(y_t^{(i)})$
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$$
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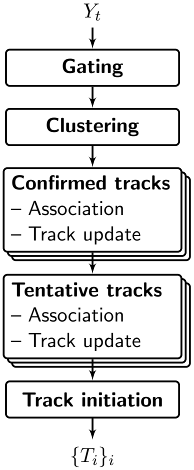
\Pr(\theta^{-1}(1) = \emptyset) &= \tfrac{1}{C}(p_{3\emptyset\emptyset} + p_{\emptyset 3\emptyset} + p_{\emptyset\emptyset\emptyset}) \\
\Pr(\theta^{-1}(3) = 3) &= \tfrac{1}{C}(p_{31\emptyset} + p_{3\emptyset 1} + p_{3\emptyset\emptyset}) \\
\Pr(\theta^{-1}(3) = 5) &= \tfrac{1}{C}(p_{\emptyset 31} + p_{\emptyset 3\emptyset}) \\
\Pr(\theta^{-1}(3) = \emptyset) &= \tfrac{1}{C}(p_{\emptyset 1\emptyset} + p_{\emptyset\emptyset 1} + p_{\emptyset\emptyset\emptyset})
\end{aligned}
$$

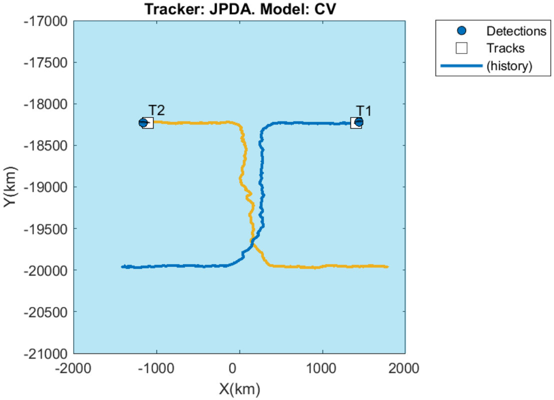
$$C = \sum_i p_i$$

Joint Probabilistic Data Association: details



- For each cluster, calculate probabilities for each target in the cluster by using a hypothesis tree.
- Use the targets PDA equivalent measurement for the update (see lecture 3).
- Unused measurements are used to initiate new tracks.
- Promote track status according to standard track logic.

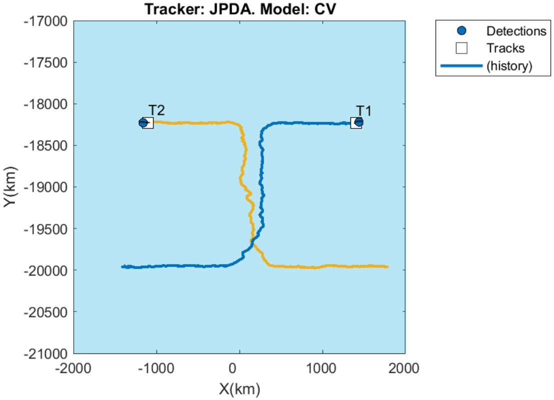
MTT: JPDA CV-model



https://youtu.be/-YB9JwiPOrY

- *Joint probabilistic data association* (JPDA) tracker
- Simple *constant velocity* (CV) model
- Note that the label switch, but there are no lost tracks.

MTT: JPDA CV-model



https://youtu.be/-YB9JwiPOrY

- *Joint probabilistic data association* (JPDA) tracker
- Simple *constant velocity* (CV) model
- Note that the label switch, but there are no lost tracks.

Joint Probabilistic Data Association: properties

- Makes no hard association decision:
  - \+ More robust in heavily cluttered environments with low P_{D}.
  - − Sub-optimal compared to using the correct associations.
- Works well when targets are well separated!
- Closely separated targets suffer from coalescence, *i.e*, neighboring tracks become identical.
- More complicated and more computationally complex than GNN.
- Consideration required when implementing the track logic.

Summary

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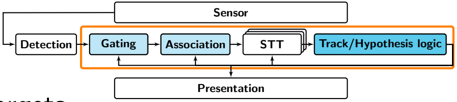
Summary

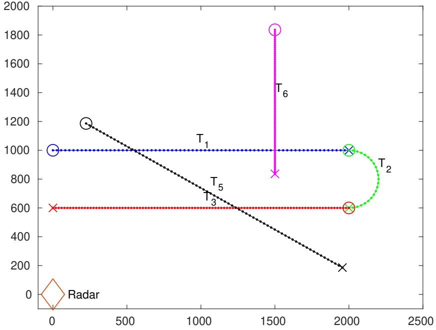


- Extended previous methods to several targets.
- Methods for gating, clustering, and association were presented, yielding the validation and association matrix.
- SHT: *One* measurement association hypothesis is used
  - GNN: A hard decision; choose the most likely association hypothesis.
    The association problem can be solved with many of-the-shelf algorithms, e.g., auction, after constructing the association (cost) matrix.
  - JPDA: A soft decision; marginalize all possible associations.
    How to combine the possible measurements depends on the association matrix.

Exercises

Exercise 2

1. Simulate a more complicated scenario, with several targets:

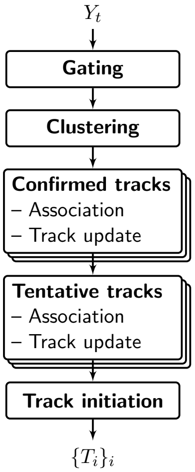


- Simulate trajectory
- Generate measurement:
  - P_D
  - P_FA
  - clutter
- Details specificed in the exercise

Exercise 2

2. MTT: GNN and JPDA

- In the exercise a detailed step-by-step instruction is given on how to build a MTT for GNN/JPDA.
- Apply the measurements to a GNN-tracker (a MATLAB version of the auction algorithm is given)
- Apply the measurements to a JPDA-tracker (MATLAB code to compute the measurement to track probabilities is available)



3. MTT: mysterious data

- At the end a mysterious data set is given without ground truth. Apply your GNN and JPDA implementations to extract the targets.

Gustaf Hendeby and Rickard Karlsson

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