A benchmark problem for robust feedback control of a flexible manipulator.

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Abstract

A benchmark problem for robust feedback control of a flexible manipulator is presented together with some suggested solutions. The system to be controlled is a four-mass system subject to input saturation, nonlinear gear elasticity, model uncertainties and load disturbances affecting both the motor and the arm. The system should be controlled by a discrete-time controller that optimizes performance for given robustness requirements.

A Benchmark Problem for Robust Feedback Control of a Flexible Manipulator

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Abstract—**A benchmark problem for robust feedback control of a flexible manipulator is presented together with some suggested solutions. The system to be controlled is a four-mass system subject to input saturation, nonlinear gear elasticity, model uncertainties and load disturbances affecting both the motor and the arm. The system should be controlled by a discrete-time controller that optimizes performance for given robustness requirements.**

Index Terms—**Robots, manipulators, flexible structures, robustness, position control**

I. Introduction

Experiments are essential in control technology research. A method that has been developed by means of realistic experiments has a larger potential to work in reality compared to methods that have not. Benchmark problems, often given in the form of mathematical equations embodied as software simulators together with performance specifications, can serve as substitute for real control experiments. A benchmark problem should be sufficiently realistic and complete but also avoid unmotivated complexity. This paper presents an industrial benchmark problem with the intention to stimulate research in the area of robust control of flexible industrial manipulators (robots). Some proposed solutions to this benchmark problem will also be presented and discussed. The authors hope that the degree of authenticity is just right and that researchers will take on the problem and eventually propose solutions and methods. When evaluating strengths and drawbacks of a certain method in the light of realistic examples, control engineers and researchers will find it easier to interpret results and the chance for the proposed method to be used in a real application increases. Thus, it is believed that the benchmark problem presented below can help increasing the ratio of control methods used outside the academic world to control methods proposed in literature. An example of a similar (at least in some parts) benchmark problem is the flexible transmission system presented by [1]. Another benchmark problem for controller design is [2] where the participants did not know the true system, which was supplied in the form of scrambled simulation code. A third example is the Grumman F-14

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Benchmark Control Problem described in [3]. However, in the area of robot manipulator control it is believed that a realistic and relevant industrial benchmark problem is needed for reasons stated above. The paper is organized as follows. Section II presents the original control problem and discusses some of the main aspects of the problem, and then in Section III the nonlinear simulation model as well as a linearized model are presented. An experimental model validation is presented in Section IV. The control design task is described in Section V, and some suggested solutions are presented in Section VI.

II. Problem Description

The most common type of industrial manipulator has six serially mounted links, all controlled by electrical motors via gears. An example of a serial industrial manipulator is shown in Figure 1.
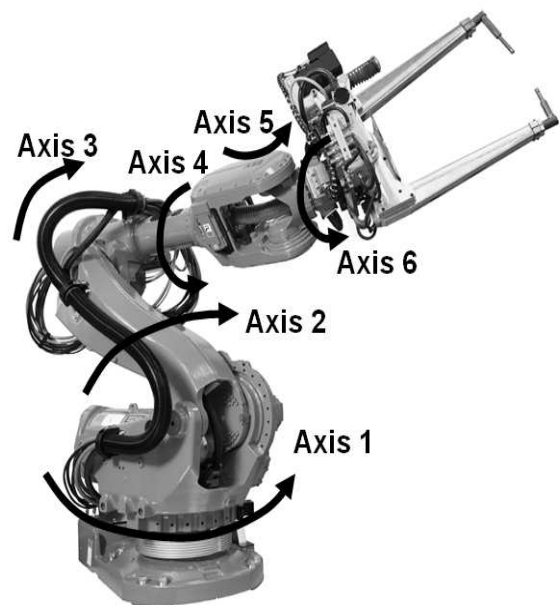

Fig. 1. IRB6600ID from ABB equipped with a spot welding gun

The dynamics of the manipulator change rapidly as the robot links move fast within its working range, and the dynamic couplings between the links are strong. Moreover, the structure is elastic and the gears have nonlinearities such as backlash, friction and nonlinear elasticity. From a control engineering perspective a manipulator can be described as a nonlinear multivariable dynamical system having the six motor currents as the inputs and the six

measurable motor angles as outputs. The goal of the motion control is to control the *orientation* and the *position* of the *tool* when moving the tool along a certain desired path.

The benchmark problem described in this paper concerns only the so-called regulator problem, where a feedback controller should be designed such that the actual tool position is close to the desired reference, in the presence of motor torque disturbances, e.g. motor torque ripple, and tool disturbances acting on the tool e.g. under material processing. For simplicity only the first axis of a horizontally mounted manipulator will be considered here. The remaining axes are positioned in a fixed configuration. In this way the influence of the nonlinear rigid body dynamics associated with the change of configuration (operating point) as well as gravity, centripetal and Coriolis torques can be neglected. Moreover, the remaining axes are positioned to minimize the couplings to the first axis. In this way, the control problem concerning the first axis can be approximated as a SISO control problem.

III. Mathematical Models

A. Nonlinear simulation model

The simulation model to be used is a four-mass model having nonlinear gear elasticity. The motor current- and torque control is assumed to be ideal so that the motor torque becomes the model input. The gearbox and motor friction effects are approximately assumed to be linear. In reality the friction normally exhibits nonlinear behavior, but, as illustrated in the model validation, the model gives a realistic description of the real system even though the friction is assumed to be linear. The model is illustrated in Figure 2. Experiments have shown that a two-mass model is not sufficient in order to describe the flexibilities. The results in [4] show that at least a three-mass model is needed in order to model the dynamics of the first axis of a moderate size robot. In [5] it can be seen than even higher model order can be necessary for some axes. Based on the authors experience, the suggested model structure is adequate for the regulator problem considered in this paper. The model structure is further justified in Section IV.
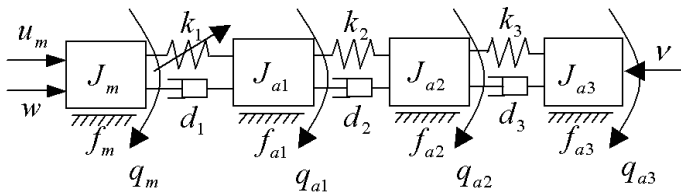
Fig. 2. Simulation model of the robot arm

The rotating masses are connected via spring-damper pairs. The first spring-damper pair, corresponding to the gear, has linear damping d_1 but nonlinear elasticity k_1. A typical relationship between deflection and torque is illustrated in Figure 3. In the simulation model the

nonlinear gear elasticity is approximated by a piecewise linear function having five segments. The second and third spring-damper pair are both assumed to be linear and represented by d_2, k_2, d_3 and k_3.
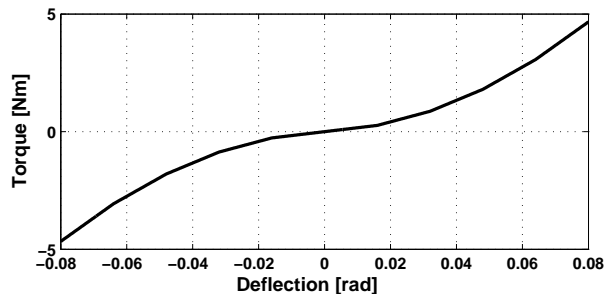
Fig. 3. Nonlinear gear elasticity: Torque as function of deflection.

The moment of inertia of the arm is here split-up into the three components J_{a1}, J_{a2} and J_{a3}. The moment of inertia of the motor is J_m. The parameters f_m, f_{a1}, f_{a2} and f_{a3} represent viscous friction in the motor and in the arm structure respectively. The motor torque u_m, which is the manipulated input of the system, is limited to ± 20 Nm. The disturbance torque acting on the motor and tool are denoted w and v respectively. The only measured output signal is the motor angle q_m, and this signal is subject to a measurement disturbance and a time delay. The variables q_{a1}, q_{a2} and q_{a3} are arm angles of the three masses and together they define the position of the tool. The angles in this model are, however, expressed on the high-speed side of the gear, so in order to get the real arm angles one must divide the model angles by the gear-ratio. Details concerning the implementation of the simulation model are given in Section V-F.

B. Linearized Model

For control design purpose there is also a linearized, with respect to the nonlinear elasticity, version of the simulation model available. The linearized model is given by

$$J\ddot{q}(t) + (D + F)\dot{q}(t) + Kq(t) = u(t) \qquad (1)$$

where

$$q(t) = \begin{bmatrix} q_m(t) & q_{a1}(t) & q_{a2}(t) & q_{a3}(t) \end{bmatrix}^T \qquad (2)$$

and

$$u(t) = \begin{bmatrix} u_m(t) + w(t) & 0 & 0 & v(t) \end{bmatrix}^T \qquad (3)$$

Furthermore

$$J = \text{diag}(J_m, J_{a1}, J_{a2}, J_{a3}) \qquad (4)$$

$$D = \begin{bmatrix} d_1 & -d_1 & 0 & 0 \\ -d_1 & d_1 + d_2 & -d_2 & 0 \\ 0 & -d_2 & d_2 + d_3 & -d_3 \\ 0 & 0 & -d_3 & d_3 \end{bmatrix} \qquad (5)$$

$$F = \text{diag}(f_m, f_{a1}, f_{a2}, f_{a3}) \qquad (6)$$

and

$$K = \begin{bmatrix} k_1 & -k_1 & 0 & 0 \\ -k_1 & k_1 + k_2 & -k_2 & 0 \\ 0 & -k_2 & k_2 + k_3 & -k_3 \\ 0 & 0 & -k_3 & k_3 \end{bmatrix} \quad (7)$$

The tool position $z(t)$ (which is the controlled variable) can for small variations around a given working point be calculated as

$$z(t) = \frac{l_1 q_{a1}(t) + l_2 q_{a2}(t) + l_3 q_{a3}(t)}{n} \quad (8)$$

where n is the gear-ratio and l_1, l_2, l_3 are distances between the (fictive) masses and the tool. Using state-space formulation the linearized system can now be described by

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (9)$$
$$y(t) = Cx(t) + n(t) \quad (10)$$
$$z(t) = Ex(t) \quad (11)$$

where $y(t)$ is the measured motor angle, $n(t)$ is measurement noise and $z(t)$ the controlled variable. Selecting the states

$$x(t) = \begin{bmatrix} q(t) \\ \dot{q}(t) \end{bmatrix} \quad (12)$$

yields

$$A = \begin{bmatrix} 0 & I \\ -J^{-1}K & -J^{-1}(D+F) \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ J^{-1} \end{bmatrix} \quad (13)$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (14)$$

and

$$E = \begin{bmatrix} 0 & \frac{l_1}{n} & \frac{l_2}{n} & \frac{l_3}{n} & 0 & 0 & 0 & 0 \end{bmatrix} \quad (15)$$

The parameter values of the nominal model, which will be denoted by M_{nom}, are defined in Table III. For the piecewise linear spring elasticity only the first segment, $k_{1,low}$, the last segment, $k_{1,high}$, and the position difference where the last segment begins, $k_{1,pos}$, are given.

IV. Model Validation

The model proposed in Section III-A is a simplification of the real problem. In order to illustrate that it, despite simplifications, is a realistic and relevant description of the problem for the purpose in this paper, some validation experiments will be presented. The model has been validated by identification and measurements on the first axis of a robot from the ABB IRB6600 series (see Figure 1) using an experimental controller. The model is semi physical, i.e. partly physical and partly gray-box with fictive physical elements. Some of the parameters were known in advance, e.g. motor inertia, total axis inertia and the nonlinear gear box elasticity. Other parameters were identified by comparing the measured frequency response of the first robot axis with the frequency response of the linear model while adjusting the parameters. Example of frequency domain identified parameters are the dampers, remaining springs and the distribution of link inertia. Finally, the distribution of the axis length parameters were adjusted to yield a similar time domain response for the tool position

when step disturbances were applied to the system during closed loop control. The resulting parameters of the validation model are close to the parameters of the benchmark model in Table III. The positions of the second and third robot links were chosen to place the tool in the middle of the working area, and the positions of the last three axes were chosen to minimize the coupling with the first axis. The frequency response was obtained by applying a multi-sine reference to a speed controller of PI type for the first axis and measuring the motor position and motor torque. The excitation energy is distributed from 3 to 30 Hz. The frequency response function (FRF) was then computed, see e.g. computation of ETFE in [6]. The FRF's for the real robot and the linear model are shown in Figure 4, and the agreement is good.
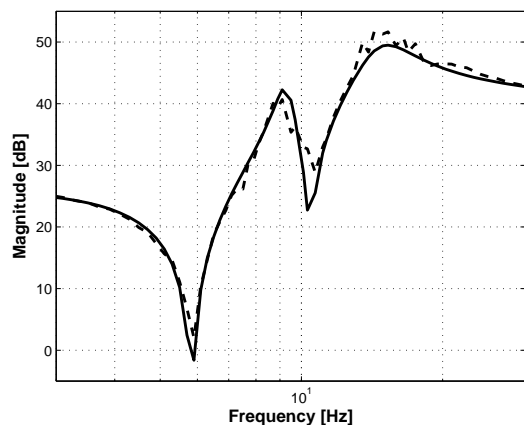

Fig. 4. The frequency response function of the linear model (solid) and of the real robot (dashed)

The first robot axis was then controlled by using a reasonably tuned PID controller of the same type as the default controller in this benchmark problem. All links were controlled with similar controllers. Torque disturbances were applied and the tool position was measured using a Leica laser measurement system LTD600 from Leica GeoSystems described in [7]. The simulation model was then simulated in closed loop with the same controller and disturbance input as in the real robot system. Figure 5 shows the tool position when a constant torque disturbance acting on the tool is suddenly released.

Figure 6 shows the tool position when applying a step in the motor torque. Note that some backlash for the real robot link is seen in the second figure.

Finally, the loop gain of the real system was increased until the stability limit was reached and the amplitude margin could be determined. The amplitude margin of the simulated system was in good correspondence with the real system. The identification and measurements show that the suggested model structure is valid for its use in this benchmark problem. The use of a SISO semi physical model and thus neglecting the interaction with other links as well as neglecting the nonlinear friction, the motor dynamics and the motor torque control is reasonably well
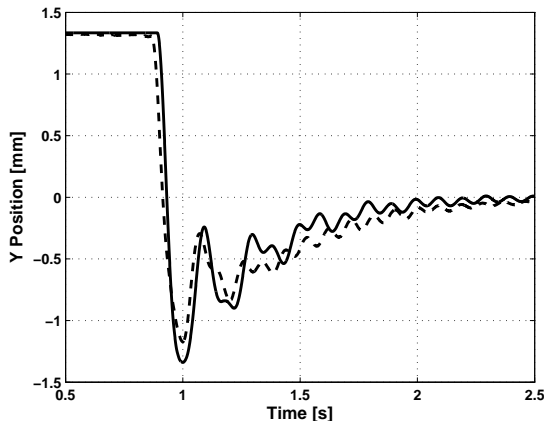
Fig. 5. The tool position for a tool step disturbance. Nonlinear simulation model (solid) and real robot (dashed)
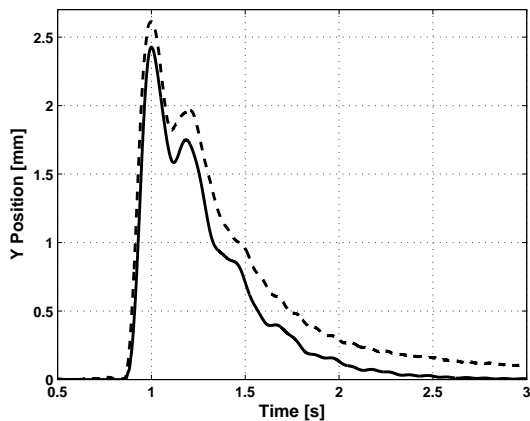

Fig. 6. The tool position for a motor step disturbance. Nonlinear simulation model (solid) and real robot (dashed)

justified.

V. The Control Design Task

A. Introduction

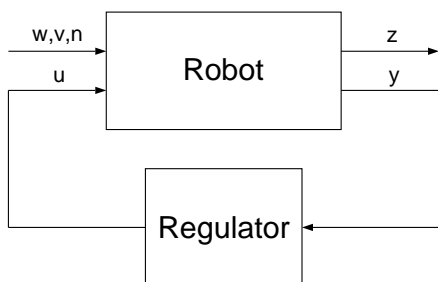The control problem can schematically be described as in Figure 7.

Fig. 7. Control system

The task can be expressed as a classical regulator problem where the aim is to reject the influence of load disturbances as much as possible, and at the same time

avoid too large input signal, and be able to handle variations in the system dynamics.

B. Load and measurement disturbances

In reality an industrial robot is affected by various disturbances. One disturbance source is the electrical motor itself which generates torque ripple, and in the model this is modeled as a load disturbance w acting on the motor, i.e. the first mass in the model. Another disturbance source is the external forces that affect the tool during e.g. material processing. This type of disturbance is modeled as a load disturbance v affecting the last mass in the model. In order to capture the various types of disturbances a specially designed sequence of disturbances will be used. It consists of torque disturbances acting on the motor and on the tool according to Figure 8, and it is a combination of steps, pulses and sweeping sinusoids (chirps). The measurement disturbance n is modeled as a band limited random noise.
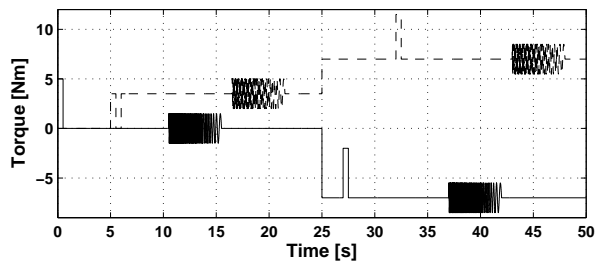

Fig. 8. Torque disturbances on motor (dashed) and tool (solid)

Figure 9 shows the tool position when the disturbance sequence in Figure 8 acts on the nominal system, and a PID-type controller is used.
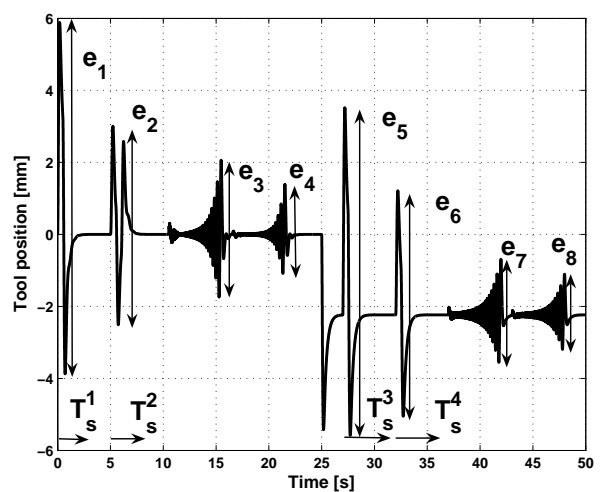

Fig. 9. Tool position when using PID-control

Figure 10 shows the input signal (motor torque) under the same conditions, and here also the influence of the measurement disturbance is evident.

The various notations in Figures 9 and 10 will be used in Section V-E, where a performance measure is formulated.
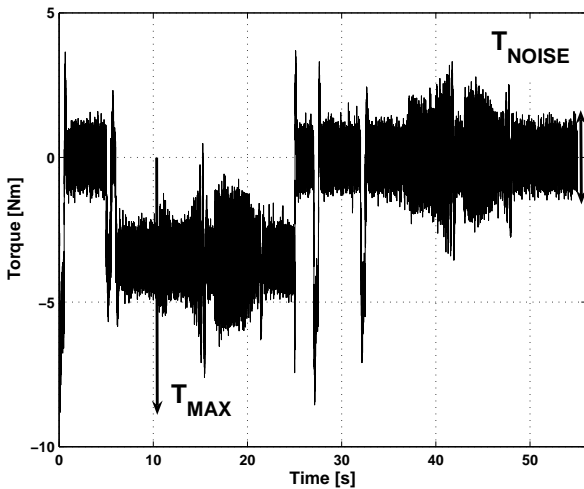
Fig. 10.   Motor torque when using PID-control

C. Parameter variations and model sets

The performance of the control systems will be evaluated for both the nominal model M_{nom} and for two sets of models which will be denoted by M_1 and M_2 respectively. In Figure 11 the frequency response function amplitude for of M_{nom} (torque to angular acceleration) is shown. The solid line corresponds to the stiffest region of the gear ($k_{1,high}$) and the dashed line corresponds to the least stiff region ($k_{1,low}$).
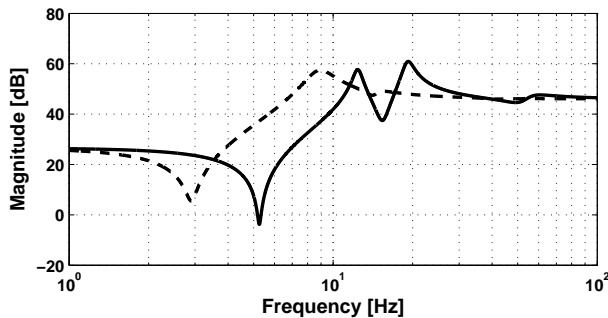

Fig. 11.   Frequency response for M_{nom}. Stiff region (solid), least stiff region (dashed).

The sets M_1 and M_2 contain ten models each. The set M_1 represents relatively small variations in the physical parameters, and the set M_2 represents relatively large variations. The Figures 12 and 13 show the absolute value of the frequency responses of the models $m \in M_1$ and $m \in M_2$ respectively, for the stiffest region of the gear ($k_{1,high}$)[1].

The uncertainty described by M_1 can be motivated by at least five sources of uncertainty:

I   **Model structure selection:** The real robot is of infinite order and the choice of model order

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[1]In the simulation model, M_1 and M_2 also have the nonlinear gear elasticity described in Section III-A.
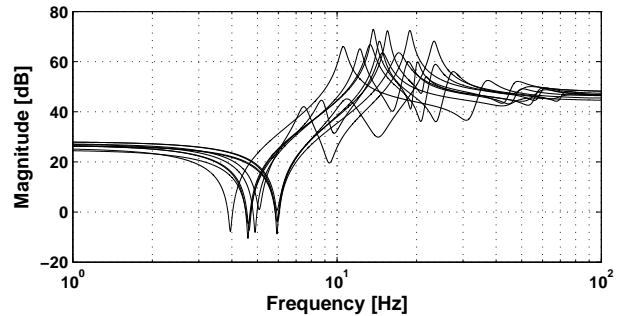

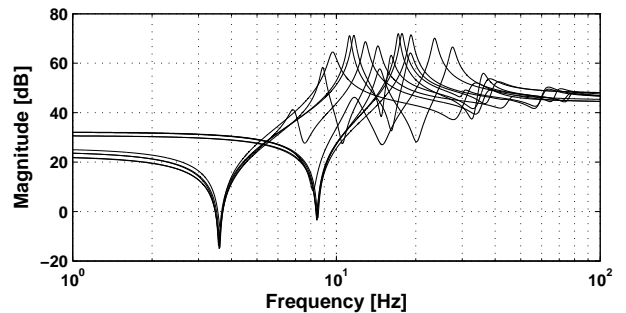Fig. 12.   Frequency response for all models in the set M_1

Fig. 13.   Frequency response for all models in the set M_2

always introduces errors. Non modeled or incompletely modeled nonlinearities such as friction and stiffness are other examples.

II   **Accuracy of nominal model parameters:** Model parameters can be obtained by identification or by other types of measurements and their values always have limited accuracy.

III   **Variation of model parameters for individual robots:** Friction and stiffness are examples of parameters that can differ significantly from one robot individual to another. Temperature dependent parameters and aging also belong to this group.

IV   **Robot installation:** The stiffness of the foundation where the robot is mounted and the user definition of tool and payload (e.g. mass and center of mass) introduces uncertainty of this type. Elasticity in the tool or payload increases the uncertainty further.

V   **Controller implementation** In a real implementation, the controller would probably be time varying by e.g. gain scheduling. Errors due to gain scheduling of controllers for different operating points also adds to the total uncertainty.

The uncertainty described by M_2 can be motivated by the fact that a real control system must be stable even for relatively large deviations between the model and the real manipulator dynamics. It is important to understand that the uncertainty is partly a design choice and depends of the actual implementation of the robot control system. One extreme is that the feedback controller has constant

parameters for all configurations and all loads, and the other is that an extremely accurate model of the robot is implemented in the robot control system. This model can then be used for gain scheduling or directly used in the feedback controller. The first extreme would have a considerably larger model set M_1 and the second extreme would have a smaller set.

D. The Design Task

The task of this benchmark problem is to minimize a performance measure by designing one or two discrete time controllers for the systems described above. The performance measure is described in Section V-E and the general requirements and some implementation constraints are described in Section V-F. One of the controllers must be capable of controlling all the models $m \in M_{nom} \cup M_1 \cup M_2$ whereas the other controller should be able to control M_{nom} alone. The controllers can be linear or nonlinear. In order to investigate how well a controller can perform when really good models are available it is recommended to design two different controllers where one is optimized for the control of M_{nom} only. This controller will in the sequel be denoted by C_1 and the other by C_2. Note that C_1 and C_2 can be identical, can have the same structure and differ only by different tuning or can have completely different structure and tuning parameters. The control requirements put on the systems in M_1 can be motivated by the fact that robust performance is required for this level of uncertainty. The use of the model set M_2 is motivated by robust stability as described above in the previous section.

E. Performance Measures

From an industrial viewpoint, time domain performance measures are to prefer when evaluating the control system. It is then a task for the control designer to translate these requirements to a form that suits the design method chosen, e.g. frequency domain norms for H_∞ design. Figures 9 and 10 show all the individual performance measures that will be weighted together into one cost function. The measures referring to the controlled output variable, tool position, are

- Peak-to-peak error (e_1 - e_8).
- Settling times (Ts_1 - Ts_4),

and the measures related to the input signal, torque, are

- Maximum value T_{MAX}
- Adjusted rms value T_{RMS}
- Torque "noise" (peak-to-peak) T_{NOISE}.

Note that T_{NOISE}, which can be caused by measurement noise and/or chattering caused by a discontinuous controller, is measured by the simulation routines when the system is at rest but that a good controller would keep the chattering/noise on a decent level also when it operates actively.

For the nominal system M_{nom} using controller C_1 the cost function V_{nom} is given by

$$V_{nom} = \gamma \sum_{i=1}^{15} \alpha_i e_i \qquad (16)$$

where e_i represents a generalized "error" (i.e. position error, settling time or torque), γ and α_i are weights. For the set M_1 using controller C_2 the maximum error from the simulations are used and the cost functions V_1 is given by

$$V_1 = \gamma \sum_{i=1}^{15} \alpha_i \max_{m \in M_1} (e_i) \qquad (17)$$

Similarly, for the set M_2 using controller C_2 the maximum error from the simulations are used and the cost functions V_2 is given by

$$V_2 = \gamma \sum_{i=1}^{15} \alpha_i \max_{m \in M_2} (e_i) \qquad (18)$$

The total cost function V is given by

$$V = \beta_{nom} V_{nom} + \beta_1 V_1 + \beta_2 V_2 \qquad (19)$$

It might then seem strange to weight performance for the set M_2 into the total cost function but it is motivated by a desire to reward the robustness of the proposed controller. It is unavoidable that this type of performance measures will be subjective, but they have been found to be relevant for a general purpose robot from an application viewpoint.

F. Implementation and Specifications

To evaluate a proposed control design a set of files has been developed. A control system in form of a Simulink^{TM}-model has been developed, and it is shown in Figure 14.
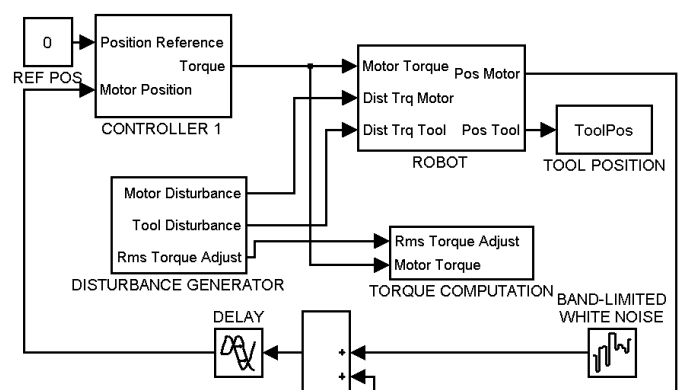

Fig. 14. The control system

In the implementation the following conditions hold:
- Sampling time 0.5 ms
- Time delay T_d 0.5 ms
- Torque saturation limits ±20 Nm (the saturation function in the controller block should not be removed)

The control task is then formulated as minimizing the overall cost function V in (19) subject to the conditions

- Settling times for M_{nom} (C_1) and M_1 (C_2): $Ts_{1,2,3,4} < 3$ s, error band $\pm$ 0.1 mm
- Settling times for M_2 (C_2): $Ts_{1,2,3,4} < 4$ s, error band $\pm$ 0.3 mm
- Torque noise $T_{NOISE} < 5$ Nm for all systems
- Stability[2] for all systems
- Stability for M_{nom} when increasing the loop gain by a factor 2.5 for C_1 and C_2
- Stability for M_{nom} when increasing the delay T_d from 0.5 ms to 2 ms for C_1 and C_2

In addition some conditions concerning the implementation have to be considered:

- Only the blocks `Controller1` and `Controller2` and the files `Controller_1.m` and `Controller_2.m` are allowed to be changed.
- No continuous-time blocks are allowed be added.
- No knowledge of deterministic nature about the noise and disturbances is allowed to be used in the controller.
- The controller C_2 must have the same initial states and parameter values for all the simulations of $m \in M_{nom} \cup M_1 \cup M_2$

The control system and the models are described in detail by Matlab$^{\text{TM}}$ and Simulink$^{\text{TM}}$ files available for download at [8]. The system comes with a simple PID-controller. The Matlab$^{\text{TM}}$ products used are described in e.g. [9].

VI. Suggested Solutions

This benchmark problem was first presented as "Swedish Open Championships in Robot Control". See [10] and [11]. On request, the benchmark problem was spread outside Sweden. The four most interesting solutions were:

A A QFT controller proposed by P.-O. Gutman, Israel Institute of Technology, Israel.
B A QFT controller of order 13 proposed by O. Roberto, Uppsala University.
C A Polynomial Controller proposed by F. Sikström and A.-K. Christiansson, University of Trollhättan/Uddevalla, Sweden.
D A so called Linear Sliding Mode Controller proposed by W.-H. Zhu, Canadian Space Agency, Canada.

For the solutions A, C, and D the controllers are of order 3 to 7. The QFT approach is generally described in [12] and the linear sliding mode approach in [13] and [14]. The polynomial controller is optimized for the given cost function and the optimization procedures used are described in [15]. The frequency responses of the controllers are shown in Figure 15 and 16. The overall shape of the frequency functions are similar with a clear lead-lag character. Solution A differs due to the peak in the magnitude curve around 10 Hz and the essentially higher high frequency (above 100 Hz) gain.

The cost function V_1 and the generalized errors for model set M_1 are shown in Table I. The table shows that

[2]The stability requirement also includes that limit cycles larger than 10 μm peak-to-peak are not allowed.
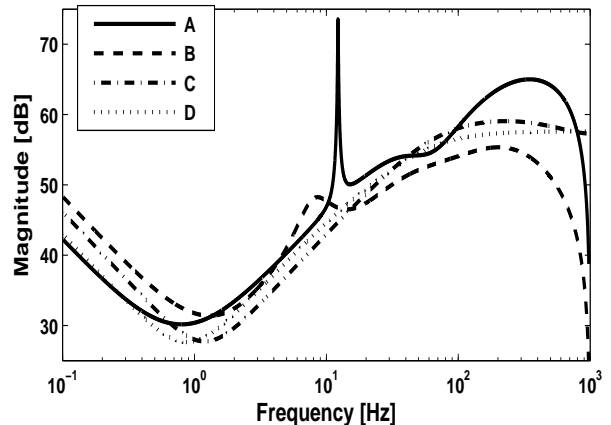

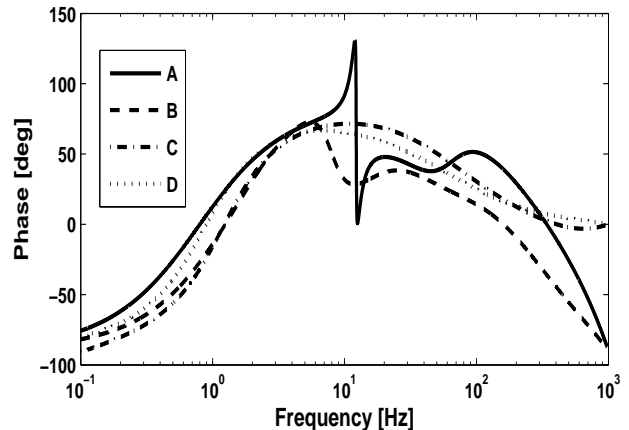Fig. 15. The absolute value of the frequency response of the controllers C_2

Fig. 16. The argument of the frequency response of the controllers C_2

no solution is in general better than all other solutions. For example, solution A gives better performance measured via the quantities e_1 to e_8 but higher values for the other measures.

TABLE I
Numerical result for model set M_1

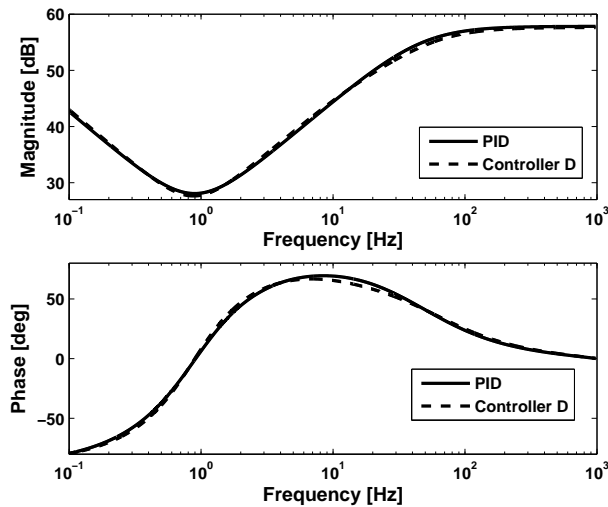| Solution	A	B	C	D
e_1 [mm]	8.22	8.57	9.75	9.11
e_2 [mm]	2.56	2.43	3.41	3.22
e_3 [mm]	5.39	5.56	5.34	5.28
e_4 [mm]	1.58	1.74	2.12	1.77
e_5 [mm]	7.78	8.22	9.37	8.64
e_6 [mm]	2.82	2.82	4.02	3.68
e_7 [mm]	4.88	5.13	4.20	4.59
e_8 [mm]	1.40	1.56	1.90	1.57
Ts_1 [s]	2.04	2.13	1.79	1.68
Ts_2 [s]	1.25	1.47	1.52	1.05
Ts_3 [s]	1.04	0.77	0.71	0.77
Ts_4 [s]	0.95	0.55	0.69	0.71
T_{NOISE} [Nm]	2.67	1.05	1.85	1.66
T_{MAX} [Nm]	12.1	12.0	11.0	11.3
T_{RMS} [Nm]	1.53	1.52	1.43	1.46
V_1	82.5	80.8	84.8	80.5

Fig. 17. The frequency response of the original controller D its PID realization

Table II shows a summary of the results. The final performance measure, defined by (19), contains weights which allow some freedom in the interpretation of the results. The weights used for computing the performance measure in Table II were selected in order to reflect good performance with respect to the original industrial problem.

TABLE II
SUMMARY OF TOTAL RESULT

Solution	A	B	C	D
V_{nom}	64.6	58.8	64.8	62.0
V_1	82.5	80.8	84.8	80.5
V_2	82.6	84.2	84.1	81.6
V	146.0	141.4	148.9	142.2

Inspection of the frequency response of controller D suggests a realization by a PID controller with derivative filter, i.e. the same structure as the default controller of the benchmark problem. In Figure 17 the frequency response of a manually tuned PID controller is compared with controller D. The performance of the PID controller is 143.4. One interesting observation is that this PID controller has complex zeros and must thus be realized as a parallel PID controller (non-interacting), see e.g. [16].

It seems hard to improve the performance further and reaching a performance index below 140 by using motor position feedback only. One possibility is to use additional or improved sensors on the motor side, e.g. speed sensors or position sensors with decreased measurement noise. Another possibility is to use additional sensors on the link side, e.g. acceleration or position sensors for the links or the tool. In [17] a performance index of 105 was reached by using tool acceleration feedback.

VII. CONCLUSIONS AND FUTURE WORK

A benchmark problem for robust control has been presented. The purpose of the problem is to formulate a problem that is industrially relevant in terms of both the system description and the performance requirements. Four proposed solutions, using different design methods, have been presented. Although the solutions use different approaches the resulting performance from all four solutions end up on the same level. For future work it would be interesting to extend the problem to a multivariable case since a real manipulator has multiple inputs and outputs. Another direction for future work could be to translate the time domain performance measures to the frequency domain, in order to enable the use of frequency domain methods for robust control design. Finally there is a potential to improve the performance by using additional sensors.

VIII. ACKNOWLEDGMENTS

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Appendix

TABLE III
Nominal parameter values

Parameter	Value	Unit
J_m	$5 \cdot 10^{-3}$	$kg \cdot m^2$
J_{a1}	$2 \cdot 10^{-3}$	$kg \cdot m^2$
J_{a2}	0.02	$kg \cdot m^2$
J_{a3}	0.02	$kg \cdot m^2$
$k_{1,high}$	100	Nm/rad
$k_{1,low}$	16.7	Nm/rad
$k_{1,pos}$	0.064	rad
k_2	110	Nm/rad
k_3	80	Nm/rad
d_1	0.08	$Nm \cdot s/rad$
d_2	0.06	$Nm \cdot s/rad$
d_3	0.08	$Nm \cdot s/rad$
f_m	$6 \cdot 10^{-3}$	$Nm \cdot s/rad$
f_{a1}	$1 \cdot 10^{-3}$	$Nm \cdot s/rad$
f_{a2}	$1 \cdot 10^{-3}$	$Nm \cdot s/rad$
f_{a3}	$1 \cdot 10^{-3}$	$Nm \cdot s/rad$
n	220	
l_1	20	mm
l_2	600	mm
l_3	1530	mm
T_d	$0.5 \cdot 10^{-3}$	s

Titel
Title   A benchmark problem for robust feedback control of a flexible manipulator.

Författare   Stig Moberg, Jonas Öhr, Svante Gunnarsson
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Sammanfattning
Abstract

A benchmark problem for robust feedback control of a flexible manipulator is presented together with some suggested solutions. The system to be controlled is a four-mass system subject to input saturation, nonlinear gear elasticity, model uncertainties and load disturbances affecting both the motor and the arm. The system should be controlled by a discrete-time controller that optimizes performance for given robustness requirements.