Homework exercise IV

LA for Filtering and Identification

Preferably hand in your solutions as a single PDF file that also includes your m-files. Each student taking part in the exam has to submit one solution set.

Exercise 1: Step input subspace identification

Consider the following SGM:

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \quad x(k) \in \mathbb{R}^n \\ y(k) &= Cx(k) + v(k) \end{aligned}$$

The input is scalar and a step input defined as,

$$u(k) = \begin{cases} 1, & k > 0 \\ 0, & k \leq 0 \end{cases}$$

The output error satisfies $v(k) \sim (0, R)$.

Method and Matlab development

1. Develop a dedicated subspace identification solution to identify $[A, B, C]$ (up to a similarity transformation) considering input-output data $\{u(k), y(k)\}_{k=0}^{N}$ by filtering this data with the filter:

   $$F(z) = (z - 1)$$

2. Develop a subspace identification method that prevents doing the filtering in item 1. For that purpose determine a projection matrix Π to cancel the input Hankel matrix $U_{0,s,N'}$ in the data equation:

   $$Y_{0,s,N'} = \mathcal{O}_s X_{0,N'} + \mathcal{T}_s U_{0,s,N'} + V_{0,s,N'}$$

   Subsequently determine how you can derive the system matrices $[A_T, B_T, C_T]$ in the limit $\lim_{N' \to \infty}$ consistently from the product $Y_{0,s,N'}\Pi$.

3. Proof the consistency or inconsistency of the subspace identification methods derived in items 1 and 2 for the given SGM.

4. Verify your results in matlab using the system and the step response data defined in the m-file `Example2.m`. Here it needs to be shown that both methods are consistent or not and that the method in item 2 gives superior results to the method in item 1. Can you explain this superiority?

5. Show mathematically that for the case process noise is present as well, both subspace identification methods are not consistent.

6. [Optional] By splitting up your data in two Hankel matrices $Y_{0,s,N'}, Y_{s,s,N'}$, and the same for the other Hankel matrices, show that the product,

   $$Y_{s,s,N'} \Pi Y_{0,s,N'}'$$

   can deliver a consistent estimate of the column space of $\mathcal{O}_s$ for the case of process- and measurement noise in the SGM.

7. [Optional] Verify the results of item 6 in matlab.

Your answer sheet needs to contain in addition to the algorithmic derivations, the m-files, the simulated and estimated step response by the different methods and the results of your experimental verification of the consistency (or inconsistency) and superiority of one method over the other.

Exercise 2: Subspace identification under periodic perturbations

Consider the SGM:

$$\begin{aligned} x(k+1) &= Ax(k) + B\Big(u(k) + d(k)\Big) \quad x(k) \in \mathbb{R}^n \\ y(k) &= Cx(k) + v(k) \end{aligned}$$

The measureable input $u(k)$ is a scalar zero-mean white noise sequence and the disturbance $d(k)$ is periodic and of the following form,

$$d(k) = \alpha \cos(\omega k + \beta)$$

where only the frequency ω is known. The additive perturbation $v(k)$ is a zero-mean white noise. Develop a subspace identification method to consistently identify up to a similarity the matrices $[A, B, C]$. Implement your solution into an m-file and test on the system in `Example3.m`, for $\omega = 0.01$ and α, β both equal to 1.

Exercise 3: The Stochastic Realization problem revisted

Let the following stochastic SGM model by given:

$$\begin{aligned} x(k+1) &= Ax(k) + Ke(k) \quad x(k) \in \mathbb{R}^n \\ y(k) &= Cx(k) + e(k) \end{aligned}$$

Assume also that $(A - KC)$ is nilpotent, such that $(A - KC)^j \equiv 0$ for $j \geq n$.

1. Show that you can express the SGM into the following VARX model:

$$y(k) = \sum_{i=1}^{n} H_i q^{-i} y(k) + e(k)$$

2. Give the explicit expression of the matrices H_i in terms of the matrices A, K, C.
3. Find a matrix $T(H_i)$ that depends on the coefficient matrices H_i, such that the row space of the matrix

$$T(H_i) Y_{k-n,n,N}$$

is the state sequence matrix $X_{k,N}$.

References

[1] M. Verhaegen and V. Verdult, “Filtering and System Identification: A Least Squares Approach”, Cambridge University Press, 2007.